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SOURCE-RELAY OPTIMIZATION FOR A TWO-WAY MIMO RELAY SYSTEM

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ABSTRACT

This paper addresses a non-regenerative two-way MIMO relay system where two sources exchange information via a common relay. Both sources and the relay are each equipped with multiple antennas. In the first phase (time or frequency), both sources transmit their signals and the relay receives. In the second phase, the relay transmits towards both sources a transformed signal. The optimization of the source covariance matrices and the optimization of the relay transformation matrix can be performed alternately until convergence. The former can be achieved by a generalized water filling algorithm, and the latter can be achieved by two algorithms discussed in this paper. The first algorithm is a hybrid gradient optimization. The second algorithm is an iterative weighted MMSE equalization. It is shown in simulation that both algorithms converge to the same results and are much faster than a prior algorithm developed for single-antenna sources.

Index Terms— Two-way relay, maximum sum rate, hybrid gradient, iterative WMMSE, generalized water-filling algorithm.

1. INTRODUCTION

Non-regenerative MIMO relays have attracted a lot of attentions recently, e.g., see [1]- [4] and the references therein. A key advantage of non-regenerative relays over regenerative relays is the simplicity and short delay. If all channel state information is available, the optimality of diagonalization for a non-regenerative MIMO relay system of any number of hops is established in [4]. A non-regenerative MIMO relay system with multiple users each with multiple antennas is addressed in a recent work [5].

In this paper, we consider a new form of non-regenerative MIMO relays where two sources each with multiple antennas exchange information via a common non-regenerative relay also with multiple antennas. Referred to as two-way MIMO relays, this problem is more complex than that in [6] where the sources and the relay are each with a single antenna. It is also more complex than that in [7] and [8] where only the

relay has multiple antennas and the two sources each have a single antenna.
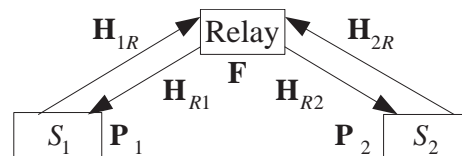
Fig. 1. A non-regenerative two-way MIMO relay system where S_1 and S_2 are the source nodes and the relay node is also referred to as R.

For this problem, we focus on the computation of the source covariance matrices and the relay transformation matrix to maximize the sum capacity of the system. We follow an approach where the source covariance matrices and the relay matrix are optimized alternately until convergence. Although the optimality of this approach is not established, there appears no other method so far that yields better results. Given any fixed relay matrix, the problem of optimizing the source covariance matrices can be done by a generalized water filling algorithm developed in [5]. For a fixed pair of source covariance matrices, we have developed two algorithms to find the optimal relay matrix. The first algorithm is a hybrid gradient method that dynamically switches between the steepest descent and the Newton's descent. Simulation shows that using the Newton's descent alone can lead to a bad convergence point. The hybrid gradient method takes the advantage of both the steepest descent and the Newton's descent. The second algorithm extends an idea discussed in [9] from broadcast beamforming to two-way MIMO relay, where the relay matrix and the linear equalizers at the two sources are optimized alternately via a weighted minimum mean square error (WMMSE) problem. The second algorithm is referred to as iterative WMMSE algorithm. Due to space limitation, we will only present the second algorithm in this paper.

Our simulation shows that both algorithms yield the same results upon convergence. However, the first algorithm converges much faster when the results are near optimal, which is expected due to the Newton descent, and the second algorithm converges much faster when the results are not near optimal. Both algorithms are useful depending on the ac-

curacy requirement. Naturally, they can be combined to achieve a better overall performance. We have also compared our algorithms with that developed in [8] for single-antenna sources. Simulation shows that both of our algorithms have much faster convergence rate than that in [8] .

The rest of this paper is organized into sections of system model and problem formulation, relay optimization with fixed sources, source optimization with fixed relay, simulation results, and conclusion.

2. SYSTEM MODEL AND PROBLEM FORMULATION

The relay system is illustrated in Fig. 1 where all channels are flat fading channels within the bandwidth of interest. The source nodes S_1 and S_2 exchange their symbol vectors $\mathbf{s}_1$ and $\mathbf{s}_2$ via the relay node R using two phases, where $\mathbf{s}_i \in \mathbb{C}^{N \times 1}$ and $\mathbf{E}[\mathbf{s}_i \mathbf{s}_i^H] = \mathbf{I}$, $i = 1, 2$. In the first phase, both the source nodes transmit, and the transmitted signal from S_i is $\mathbf{x}_i = \mathbf{P}_i \mathbf{s}_i$, where $\mathbf{P}_i \in \mathbb{C}^{N \times N}$ is the transmit beamforming matrix from S_i, and $\mathbf{R}_{x_i} \doteq \mathbf{P}_i \mathbf{P}_i^H$ is the source covariance matrix from S_i. Also in the first phase, the relay receives $\mathbf{H}_{1R} \mathbf{x}_1 + \mathbf{H}_{2R} \mathbf{x}_2 + \mathbf{n}_R$ where $\mathbf{n}_R \in \mathbb{C}^{M \times 1}$ is the noise at the relay and we assume $\mathbf{n}_R \sim \mathcal{CN}(0, \mathbf{I})$. In the second phase, the relay uses the transformation matrix $\mathbf{F} \in \mathbb{C}^{M \times M}$ and transmits $\mathbf{x}_R = \mathbf{F} \mathbf{H}_{1R} \mathbf{x}_1 + \mathbf{F} \mathbf{H}_{2R} \mathbf{x}_2 + \mathbf{F} \mathbf{n}_R$ where $\mathbf{H}_{iR} \in \mathbb{C}^{M \times N}$ denotes the channel matrix from S_i to R. The power consumed at the relay is

$$p_R = \text{Tr}(\mathbf{F} \mathbf{H}_{1R} \mathbf{R}_{x_1} \mathbf{H}_{1R}^H \mathbf{F}^H + \mathbf{F} \mathbf{H}_{2R} \mathbf{R}_{x_2} \mathbf{H}_{2R}^H \mathbf{F}^H + \mathbf{F} \mathbf{F}^H).$$

Also in the second phase, S_i receives $\mathbf{H}_{Ri} \mathbf{x}_R + \mathbf{n}_i$ where $\mathbf{H}_{Ri} \in \mathbb{C}^{N \times M}$ is the channel matrix from R to S_i and $\mathbf{n}_i \in \mathbb{C}^{N \times 1}$ is the noise at S_i. We also assume $\mathbf{n}_i \sim \mathcal{CN}(0, \mathbf{I})$. Since S_i knows $\mathbf{H}_{Ri} \mathbf{F} \mathbf{H}_{iR} \mathbf{x}_i$ (which is a feasible assumption), S_i can subtract it out from $\mathbf{H}_{Ri} \mathbf{x}_R + \mathbf{n}_i$ and the net signals received at S_1 and S_2 are

$$\mathbf{y}_1 = \mathbf{H}_{R1} \mathbf{F} \mathbf{H}_{2R} \mathbf{x}_2 + \mathbf{H}_{R1} \mathbf{F} \mathbf{n}_R + \mathbf{n}_1 \tag{1}$$

$$\mathbf{y}_2 = \mathbf{H}_{R2} \mathbf{F} \mathbf{H}_{1R} \mathbf{x}_1 + \mathbf{H}_{R2} \mathbf{F} \mathbf{n}_R + \mathbf{n}_2 \tag{2}$$

The maximum achievable rates in bits/s/Hz: r_{21} from S_2 to S_1, and r_{12} from S_1 to S_2, can be expressed as $r_{21} = \frac{1}{2} \log_2 \det \mathbf{X}_1$ and $r_{12} = \frac{1}{2} \log_2 \det \mathbf{X}_2$ where

$$\mathbf{X}_1 = \mathbf{I} + \mathbf{R}_{x_2}^{\frac{H}{2}} \mathbf{H}_{2R}^H \mathbf{F}^H \mathbf{H}_{R1}^H$$
$$(\mathbf{H}_{R1} \mathbf{F} \mathbf{F}^H \mathbf{H}_{R1}^H + \mathbf{I})^{-1} \mathbf{H}_{R1} \mathbf{F} \mathbf{H}_{2R} \mathbf{R}_{x_2}^{\frac{1}{2}} \tag{3}$$

and $\mathbf{X}_2$ has the same form as $\mathbf{X}_1$ but with the indices 1 and 2 exchanged. The source-relay optimization problem for the two-way relay system is now formulated as

$$\min_{\mathbf{F}, \mathbf{R}_{x_i}} -r_{sum} \doteq -r_{21} - r_{12} \tag{4}$$

s.t. $p_R \leq P_R$ and $\text{Tr}(\mathbf{R}_{x_i}) \leq P_i$, $i = 1, 2$, where P_R and P_i denote the maximum available power at R and S_i. Weighted sum rate can be treated similarly. We will follow the approach that finds the optimal $\mathbf{F}$ and the optimal $\mathbf{R}_i$ for $i = 1, 2$ alternately until convergence.

3. RELAY OPTIMIZATION WITH FIXED SOURCE

In this section, we consider the optimization of $\mathbf{F}$ with fixed $\mathbf{R}_i$ for $i = 1, 2$.

3.1. Optimal Structure of the Relay Matrix

Theorem 3.1 in [8] is valid for sources each with a single antenna. The following is a generalization of that theorem from single antenna to multiple antennas at the source nodes. Assuming $M \geq 2N$ and using QR decomposition, we can obtain that $\begin{bmatrix} \mathbf{H}_{R1}^H & \mathbf{H}_{R2}^H \end{bmatrix} = \mathbf{V}_1 \mathbf{R}_1$ and $\begin{bmatrix} \mathbf{H}_{1R} & \mathbf{H}_{2R} \end{bmatrix} = \mathbf{U}_2 \mathbf{R}_2$ where $\mathbf{R}_1, \mathbf{R}_2 \in \mathbb{C}^{2N \times 2N}$ are triangular, and $\mathbf{V}_1, \mathbf{U}_2 \in \mathbb{C}^{M \times 2N}$ are orthonormal. Then, we have the following theorem

Theorem 1. *If $M \geq 2N$, the optimal relay matrix $\mathbf{F}$ that maximizes the sum rate r_{sum} has the following structure:*

$$\mathbf{F} = \mathbf{V}_1 \mathbf{A} \mathbf{U}_2^H \tag{5}$$

where $\mathbf{A} \in \mathbb{C}^{2N \times 2N}$

The proof is available in [10]. Thus, if the condition $M \geq 2N$ is satisfied, we only need to determine $\mathbf{A}$ with $4N^2$ unknown elements instead of M^2 in $\mathbf{F}$. This can greatly reduce the computational load. To unify the structure of the problem regardless of whether $M \geq 2N$ is satisfied, we will write

$$\mathbf{F} = \mathbf{S} \mathbf{A} \mathbf{T}^H \text{where} \begin{cases} \mathbf{S} = \mathbf{V}_1, \mathbf{T} = \mathbf{U}_2 & \text{if } M > 2N \\ \mathbf{S} = \mathbf{T} = \mathbf{I} & \text{if } M \leq 2N \end{cases} \tag{6}$$

also define $\mathbf{G}_{R1} = \mathbf{H}_{R1} \mathbf{S}$, $\mathbf{G}_{R2} = \mathbf{H}_{R2} \mathbf{S}$, $\mathbf{G}_{1R} = \mathbf{T}^H \mathbf{H}_{1R}$, $\mathbf{G}_{2R} = \mathbf{T}^H \mathbf{H}_{2R}$. Then, the signal models become

$$\mathbf{y}_1 = \mathbf{G}_{R1} \mathbf{A} \mathbf{G}_{2R} \mathbf{x}_2 + \mathbf{G}_{R1} \mathbf{A} \mathbf{n}_R' + \mathbf{n}_1 \tag{7}$$

$$\mathbf{y}_2 = \mathbf{G}_{R2} \mathbf{A} \mathbf{G}_{1R} \mathbf{x}_1 + \mathbf{G}_{R2} \mathbf{A} \mathbf{n}_R' + \mathbf{n}_2 \tag{8}$$

where $\mathbf{n}_R' = \mathbf{T}^H \mathbf{n}_R \sim \mathcal{CN}(0, \mathbf{I})$. By comparing (1)-(2) with (7)-(8), we can see that the problem structure without the use of the optimal structure of $\mathbf{F}$ is the same as that with its use although there is a reduction of the problem dimension in the latter case.

3.2. Iterative WMMSE Algorithm

The Lagrangian function of the optimization problem (4) with respect to $\mathbf{A}$ (or equivalently $\mathbf{F}$) only is

$$L_1(\mathbf{A}) = -\frac{1}{2} \log_2 \det \mathbf{X}_1 - \frac{1}{2} \log_2 \det \mathbf{X}_2 + \eta(p_R - P_R) \tag{9}$$

where $\eta \in \mathbb{R}$ and $\eta \geq 0$. The differential of $L_1(\mathbf{A})$ can be expressed by

$$\partial L_1(\mathbf{A}) = -\frac{1}{2\ln 2}\text{Tr}(\mathbf{X}_1^{-1}\partial\mathbf{X}_1) - \frac{1}{2\ln 2}\text{Tr}(\mathbf{X}_2^{-1}\partial\mathbf{X}_2) \\ + \partial(\eta(p_R - P_R)) \quad (10)$$

We now consider the following problem

$$\min_{\mathbf{A}} \quad \text{Tr}(\mathbf{W}_1\mathbf{X}_1^{-1}) + \text{Tr}(\mathbf{W}_2\mathbf{X}_2^{-1}) \quad (11)$$

s.t. $p_R \leq P_R$, where $\mathbf{W}_i$, $i = 1, 2$, are fixed weighting matrices. It is known that $\mathbf{X}_1^{-1}$ is the covariance matrix of the linear MMSE estimate of $\mathbf{s}_2$ using $\mathbf{y}_1$, and $\mathbf{X}_2^{-1}$ is the covariance matrix of the linear MMSE estimate of $\mathbf{s}_1$ using $\mathbf{y}_2$. So, the problem (11) is referred to as WMMSE (weighted MMSE) problem. The differential of the Lagrangian function $L_2(\mathbf{A})$ of the problem (11) can be expressed by

$$\partial L_2(\mathbf{A}) = -\text{Tr}(\mathbf{X}_1^{-1}\mathbf{W}_1\mathbf{X}_1^{-1}\partial\mathbf{X}_1) \\ - \text{Tr}(\mathbf{X}_2^{-1}\mathbf{W}_2\mathbf{X}_2^{-1}\partial\mathbf{X}_2) + \partial(\nu(p_R - P_R)) \quad (12)$$

where $\nu \in \mathbb{R}$, $\nu \geq 0$.

By comparing (10) with (12), we see that the problem (4) with respect to $\mathbf{A}$ only is equivalent to the WMMSE problem (11) if

$$\mathbf{W}_1 = \frac{1}{2\ln 2}\mathbf{X}_1 \quad \text{and} \quad \mathbf{W}_2 = \frac{1}{2\ln 2}\mathbf{X}_2 \quad (13)$$

Now, given any choice of $\mathbf{A}$, it is a standard task to determine the linear MMSE equalizers (receivers): $\mathbf{Q}_1$ of $\mathbf{y}_1$, and $\mathbf{Q}_2$ of $\mathbf{y}_2$, and easy to determine $\mathbf{W}_1$ and $\mathbf{W}_2$.

With any given $\mathbf{Q}_i$ and $\mathbf{W}_i$ for $i = 1, 2$, the WMMSE problem (11) becomes equivalent to

$$\min_{\mathbf{A}} \quad J \doteq \text{Tr}\Big(\mathbf{W}_1\mathrm{E}\big[(\mathbf{Q}_1\mathbf{y}_1 - \mathbf{s}_2)(\mathbf{Q}_1\mathbf{y}_1 - \mathbf{s}_2)^H\big]\Big) \\ + \text{Tr}\Big(\mathbf{W}_2\mathrm{E}\big[(\mathbf{Q}_2\mathbf{y}_2 - \mathbf{s}_1)(\mathbf{Q}_2\mathbf{y}_2 - \mathbf{s}_1)^H\big]\Big) \quad (14)$$

s.t. $p_R \leq P_R$, which can be used to find the optimal $\mathbf{A}$. More specifically, by inserting (1) and (2) into (14), we can show that the problem (14) is equivalent to

$$\min_{\mathbf{a}} \quad J = \mathbf{a}^H\mathbf{S}\mathbf{a} - \mathbf{c}^H\mathbf{a} - \mathbf{a}^H\mathbf{c} + d \quad (15)$$

s.t. $\mathbf{a}^H\mathbf{S}_R\mathbf{a} - P_R \leq 0$, where $\mathbf{a} = \text{vec}(\mathbf{A})$, $\mathbf{S}_R = (\mathbf{G}_{1R}\mathbf{R}_{x_1}\mathbf{G}_{1R}^H + \mathbf{G}_{2R}\mathbf{R}_{x_2}\mathbf{G}_{2R}^H + \mathbf{I})^* \otimes \mathbf{I}$, $\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$, $\mathbf{c} = \mathbf{c}_1 + \mathbf{c}_2$, $d = d_1 + d_2$, $\mathbf{S}_i = (\mathbf{G}_{i^*R}\mathbf{R}_{x_i^*}\mathbf{G}_{i^*R}^H)^* \otimes (\mathbf{G}_{Ri}^H\mathbf{Q}_i^H\mathbf{W}_i\mathbf{Q}_i\mathbf{G}_{Ri}) + \mathbf{I} \otimes (\mathbf{G}_{Ri}^H\mathbf{Q}_i^H\mathbf{W}_i\mathbf{Q}_i\mathbf{G}_{Ri})$, $\mathbf{c}_i = \text{vec}(\mathbf{G}_{Ri}^H\mathbf{Q}_i^H\mathbf{W}_i\mathbf{R}_{x_i^*}^{\frac{H}{2}}\mathbf{G}_{i^*R}^H)$, and $d_i = \text{Tr}(\mathbf{W}_i\mathbf{Q}_i\mathbf{Q}_i^H) + \text{Tr}(\mathbf{W}_i)$. Here, we have used $i^* \doteq (i - 1)_{\text{mod } 2} + 1$. The problem (15) is a simple convex problem.

Therefore, the iterative WMMSE algorithm is: 1) find $\mathbf{Q}_i$ and $\mathbf{W}_i$ for $i = 1, 2$ assuming a fixed $\mathbf{A}$; 2) find a new $\mathbf{A}$ by fixing the previously found $\mathbf{Q}_i$ and $\mathbf{W}_i$ for $i = 1, 2$; and 3) go back to step 1) until convergence.

4. SOURCE OPTIMIZATION WITH FIXED RELAY

In this section, we consider the optimization of $\mathbf{R}_{x_1}$ and $\mathbf{R}_{x_2}$ with a fixed $\mathbf{F}$. We can rewrite the problem (4) with respect to $\mathbf{R}_{x_1}$ and $\mathbf{R}_{x_2}$ only as follows:

$$\min_{\mathbf{R}_x \succeq 0} \quad -\log_2\det(\mathbf{I} + \mathbf{H}\mathbf{R}_x\mathbf{H}^H)$$
$$s.t. \quad \text{Tr}(\mathbf{B}_i\mathbf{R}_x\mathbf{B}_i^H) \leq P_i, \quad i = 1, 2, 3 \quad (16)$$

where $\mathbf{R}_x = \text{diag}(\mathbf{R}_{x_1}, \mathbf{R}_{x_2})$, $\mathbf{H} = \text{diag}(\mathbf{H}_1, \mathbf{H}_2)$, $\mathbf{B}_1 = \text{diag}(\mathbf{I}, \mathbf{0})$, $\mathbf{B}_2 = \text{diag}(\mathbf{0}, \mathbf{I})$, $\mathbf{B}_3 = \text{diag}(\mathbf{G}_1, \mathbf{G}_2)$, $\mathbf{H}_1 = (\mathbf{H}_{1R}^H\mathbf{F}^H\mathbf{H}_{R2}^H(\mathbf{H}_{R2}\mathbf{F}\mathbf{F}^H\mathbf{H}_{R2}^H + \mathbf{I})^{-1}\mathbf{H}_{R2}\mathbf{F}\mathbf{H}_{1R})^{\frac{1}{2}}$, $\mathbf{H}_2 = (\mathbf{H}_{2R}^H\mathbf{F}^H\mathbf{H}_{R1}^H(\mathbf{H}_{R1}\mathbf{F}\mathbf{F}^H\mathbf{H}_{R1}^H + \mathbf{I})^{-1}\mathbf{H}_{R1}\mathbf{F}\mathbf{H}_{2R})^{\frac{1}{2}}$, $\mathbf{G}_1 = \mathbf{F}\mathbf{H}_{1R}$, $\mathbf{G}_2 = \mathbf{F}\mathbf{H}_{2R}$, and $P_3 = P_R - \text{Tr}(\mathbf{F}\mathbf{F}^H)$. According to Theorem 1 in [5], we can use the generalized water-filling algorithm to obtain the solution to (16).

Note that if we allow $\mathbf{R}_x$ to be an arbitrary positive-semidefinite matrix, we can show that the solution to the problem (16) is always such that $\mathbf{R}_x$ is block diagonal. Assume $\mathbf{R}_x = \begin{bmatrix} \mathbf{R}_{x_1} & \mathbf{R} \\ \mathbf{R}^H & \mathbf{R}_{x_2} \end{bmatrix}$. Using the Fischer's inequality, we have

$$\det(\mathbf{I} + \mathbf{H}\mathbf{R}_x\mathbf{H}^H) \leq \det(\mathbf{I} + \mathbf{H}\text{diag}(\mathbf{R}_{x_1}, \mathbf{R}_{x_2})\mathbf{H}^H) \quad (17)$$

which says that the off-diagonal blocks of $\mathbf{R}_x$ can only reduce the sum rate. Furthermore, we can write

$$\text{Tr}(\mathbf{B}_i\mathbf{R}_x\mathbf{B}_i^H) = \text{Tr}(\mathbf{B}_i\text{diag}(\mathbf{R}_{x_1}, \mathbf{R}_{x_2})\mathbf{B}_i^H), \; i = 1, 2, 3 \quad (18)$$

which says that the off-diagonal blocks of $\mathbf{R}_x$ do not affect the power constraints. Therefore, the optimal solution of the problem (16) must be such that $\mathbf{R}_x$ is block diagonal.

5. SIMULATION RESULTS

We measure the performance of an algorithm by its time-capacity curve. A point on the curve corresponds to the time required to obtain the corresponding achievable capacity. It is important to note for any given set of source covariance matrices and the relay matrix, there is a corresponding achievable capacity. With a properly placed check point, any optimization algorithm or program can iteratively generate a sequence of these sets of matrices, corresponding to a sequence of values of achievable capacity, and also another sequence of time stamps. These two sets of sequences can be gathered via simulation and illustrated as a time-capacity curve.
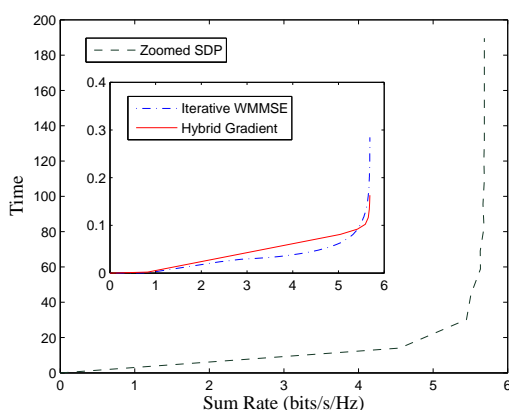
We now present a comparison of the time-capacity curves of the three algorithms for relay optimization: iterative WMMSE, hybrid gradient and zoomed SDP. The zoomed SDP algorithm is a fast variation of the algorithm developed in [8] for single antenna sources. The details of both hybrid gradient and zoomed SDP are available in [10]. We assume $N = 1$ and $M = 5$ for this comparison.

Fig. 2 compares the time-capacity curves of the three algorithms using an arbitrary realization of all channel matrices. All elements of the channel matrices $\mathbf{H}_{1R}$, $\mathbf{H}_{2R}$, $\mathbf{H}_{R1}$ and $\mathbf{H}_{R2}$ were randomly generated according to $\mathcal{CN}(0,1)$. This figure shows that iterative WMMSE is much faster than hybrid gradient when the capacity is not near optimal, but the opposite is true when the capacity is near optimal. This figure also shows that the zoomed SDP algorithm is much slower than both iterative WMMSE and hybrid gradient.

For Fig. 3, we assumed $N = 2$ antennas at each of the two source nodes and $M = 6$ antennas at the relay node. This figure compares the final (i.e., after convergence) achievable capacities under four schemes: no optimization, source optimization only, relay optimization only, and joint source-relay optimization. When not optimized, the source covariance matrices were randomly generated, and so was the relay matrix except that the structure in (5) was always used. In this example, the relay optimization beyond (5) appears more effective than the source optimization.

6. CONCLUSION

We have presented a technique for joint source-relay optimization for a non-regenerative two-way MIMO relay system. The source covariance matrices and the relay matrix are optimized alternately until convergence. The source optimization with fixed relay follows a generalized water filling algorithm developed in [5]. For the relay optimization with fixed source, we have developed a novel algorithm called iterative WMMSE algorithm. This algorithm is shown in simulation to have a faster convergence rate than a hybrid gradient algorithm when the results are not near optimal, but a slower convergence rate when the results are near optimal. However, both of the new algorithms are much faster than a prior algorithm developed in [8] for single antenna sources.

Fig. 2. A comparison of time-capacity curves of iterative WMMSE, hybrid gradient and zoomed SDP. The time was measured in seconds by running the three algorithms on a desktop PC.
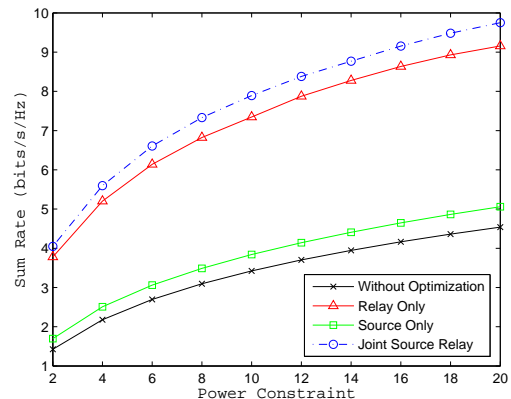
Fig. 3. Comparison of averaged sum capacities under four schemes: no optimization, source optimization only, relay optimization only, and joint source-relay optimization. The average was done using 50 randomly generated channels. Here, $P_1 = P_2 = P_R = P \in [2, 20]$.

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